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ÉCOLE POLYTECHNIQUE



CENTRE DE RECHERCHE SUR LES TRANSPORTS
UNIVERSITÉ DE MONTRÉAL
C.P. 6128, SUCCURSALE A,
MONTRÉAL
CANADA H3C 3J7

TÉLÉPHONE: (514) 343-7575
TÉLEX: 05-24146 BIBPOLYTEC
EMAIL: crt@crt.umontreal.ca
TÉLÉCOPIEUR (FAX): (514) 343-7121

CRT-692

A DECISION-SUPPORT SYSTEM FOR
PAVEMENT MANAGEMENT AT THE NETWORK LEVEL

by

Michel Gendreau
Louis-Philippe Duclos

This paper was presented at the 69th Transportation Research Board Meeting in Washington, January 7-11, 1990.

An earlier version of this paper was published in the Proceedings of the 5th World Conference on Transportation Research (Yokohama, Japan, July 10-14, 1989).

Centre de recherche sur les transports - Publication #692

May 1990

Dépôt légal - 2e trimestre 1990

ABSTRACT

Over the recent years, most agencies responsible for the management of large highway and road networks have been putting an increasing emphasis on pavement management issues. With highway networks which are ageing and deteriorating rapidly, these agencies are forced to undertake a rigorous planning exercise, if the scarce resources which are available for pavement maintenance and rehabilitation are to be allocated in an efficient fashion.

To improve their planning mechanisms, many agencies have resorted to pavement management systems built around optimization models of one type or another. The general tendency in these systems has been to provide planners with a single well-defined optimization model, i.e., with a model in which the objective function and the constraints have been stated once and for all. When the complexity of the issues and of the trade-offs involved in pavement management is considered, one is forced to conclude that this single model approach falls short of fulfilling the needs of decision makers: there are always important questions that remain unanswered. To remedy this situation, we propose a different approach: provide planners with a flexible decision-support tool which lets them specify themselves the optimization problems which seem meaningful within the scope of the decision-making process at any given time.

The core of the proposed decision-support system is a general Markov Decision Process structure in which administrative divisions of the highway network (regions, districts) are explicitly taken into account. This basic structure is stated in an equivalent linear programming format which then allows for the addition to the model of various linear constraints.

The other fundamental element of the system is a formal descriptive framework which is made up of two languages:

- a **network language** which is used to describe the condition states, the administrative structure, the highway categories, the transition matrices describing pavement evolution through time, the maintenance and rehabilitation actions, and various linear functions defined over the set of condition states or actions (performance ratings, costs, etc.);
- a **scenario language** which allows a planner, who is unfamiliar with optimization models and techniques, to define the optimization problem that he or she wants to have solved with only a few statements.

In this paper, we provide a description of both languages, of the general Markov Decision Process structure, and of the current computer implementation of the system. Planned future developments are also outlined.

Key words: pavement management; planning; decision-support; optimization.

RÉSUMÉ

Nous présentons un système d'aide à la décision pour la planification à moyen et long terme des interventions d'entretien et de réhabilitation des chaussées sur un réseau routier de grande taille.

Le modèle mathématique sur lequel repose ce système est un modèle de décision markovien avec contraintes supplémentaires, semblable aux modèles utilisés dans les systèmes de gestion des chaussées de plusieurs états américains; notre modèle diffère cependant de ces modèles par le fait qu'il permet de tenir compte explicitement de la structure administrative (découpage en régions et districts) du réseau étudié.

L'autre élément distinctif du système proposé est un cadre descriptif formel constitué de deux langages :

- 1) **Un langage de description de réseaux** que le planificateur doit utiliser pour spécifier les paramètres du réseau étudié : la structure administrative (régions, districts), les catégories de routes (autoroutes, routes principales, secondaires, etc.), les états possibles des diverses chaussées, les interventions permises en fonction des états, les fonctions caractérisant les états et les interventions (indicateurs de performance, coûts, etc.), la matrice de transition décrivant l'évolution des chaussées dans le temps, l'état initial du réseau.
- 2) **Un langage de description de scénarios** permettant à un usager non familier avec la programmation mathématique de spécifier en quelques énoncés le problème particulier qui l'intéresse en termes de réseau à considérer, d'horizon de planification, d'objectif à optimiser, de contraintes à respecter et d'état-cible à atteindre à la fin de l'horizon.

Dans cet article, nous présentons ces deux langages, le modèle de décision markovien et l'implantation informatique actuelle du système. Nous indiquons aussi les développements futurs prévus.

Mots-clés : entretien routier; réhabilitation des chaussées; planification; modèle mathématique; optimisation; système d'aide à la décision.

1. INTRODUCTION

Over the recent years, most agencies responsible for the management of large highway and road networks have been putting an increasing emphasis on pavement management issues. With highway networks which are ageing and deteriorating rapidly, these agencies are forced to undertake a rigorous planning exercise, if the scarce resources which are available for pavement maintenance and rehabilitation are to be allocated in an efficient fashion.

To improve their planning mechanisms, many agencies have resorted to pavement management systems (PMSs) built around optimization models of one type or another. A non-exhaustive list of these agencies includes the departments of transportation of the states of Arizona (4, 11), Florida (8), and Texas (1) in the United States, the ministries of transportation of the provinces of Ontario (3) and Manitoba (5) in Canada, of Denmark (10) and Finland (9), as well as the World Bank (7).

The general tendency in these systems has been to provide planners with a single well-defined optimization model, i.e., with a model in which the objective function and the constraints have been stated once and for all. When the complexity of the issues and of the trade-offs involved in pavement management is considered, one is forced to conclude that this single model approach falls short of fulfilling the needs of decision makers: there are always important questions that remain unanswered. To remedy this situation, we propose a different approach: provide planners with a flexible decision- support tool which lets them specify themselves the optimization problems which seem meaningful within the scope of the decision-making process at any given time.

It should be noted that, on our part, the decision to adopt this decision-support system (DSS) approach for pavement management planning was arrived at after a series of meetings and discussions with officials of the

Ministry of Transportation of the province of Quebec in Canada. Throughout these discussions, they stressed the multi-objective nature of pavement management and they clearly indicated the need of having a flexible computerized planning tool for addressing their planning problems.

To clearly illustrate how the DSS approach can work for pavement management, we describe in this paper a specific DSS which focusses on **strategic planning** (also called network level planning by some authors) issues in large highway networks, i.e., allocation of budgets to various highway categories and to administrative units (regions, districts), setting of priorities and performance objectives, evaluation of major rehabilitation or maintenance programs, etc. over a medium to long-term planning horizon. This system is intended to be integrated with a **tactical planning** (or programming) system to form a truly comprehensive pavement management system along the lines proposed by Gendreau (2).

The two key "elements" of the proposed DSS are a **basic optimization framework** (a generic optimization model) and a **formal descriptive environment** for pavement management planning problems. The basic optimization framework provides decision-makers with a "skeleton" optimization model which they can "flesh out" by using the formal descriptive environment.

In the remainder of the paper, we first describe the key elements of the DSS, we then provide some examples of problem description within the system and discuss the computer implementation of the system; a short discussion of possible future developments and improvements concludes the paper.

2. THE BASIC OPTIMIZATION FRAMEWORK

When one is faced with the task of choosing an optimization framework around which a DSS will be built, it is of prime importance to properly assess the objectives of the envisioned system. In the case under consideration here the intended use of the system is, as we stated earlier, strategic planning for large highway networks. For such purposes, it seems preferable to choose an aggregate model, (i.e., a model in which pavements are grouped together according to some criterion), rather than a disaggregate one (i.e., a model in which individual highway sections are explicitly identified), since this will ease the computational burden (this may be a critical factor, if the network is very large) and will also help decision makers to focus their attention on critical issues (see (2, 3) for a more detailed discussion on this topic).

Amongst the aggregate optimization formulations for pavement management planning, the most popular are undoubtedly those of the **Markov Decision Process (MDP)** type, which were first introduced in the Network Optimization System of the state of Arizona (4, 11). In those models, the grouping criterion for pavements is their **condition state** (a combination of characteristics or attributes relative to their current condition and possibly their dynamic behavior), and pavement evolution over time, which corresponds to transitions between condition states, is described in a probabilistic fashion.

An interesting feature of MDP type models is that they can be reformulated in a linear programming (LP) format. This reformulation is advantageous in many ways: (i) the model may be easily solved using standard LP packages; (ii) arbitrary linear constraints may be freely added to the LP formulation to account for other aspects of the planning problem (it should be pointed out however that adding such constraints to the model "destroys" its MDP nature from a formal viewpoint).

With these advantages in mind, it was thus decided to choose an MDP with additional linear constraints structure for the optimization framework. The specific MDP model which is the core of the framework differs however from previous models in that administrative divisions of the highway network (regions, districts) are explicitly taken into account in the formulation. The reasons for this are two-fold:

- (i) This model will allow for a much easier interfacing between the DSS and the future tactical planning system (see (2) for further details).
- (ii) This model provides decision makers with a much richer environment for problem statement, since it makes it possible to define constraints relative to the administrative divisions of the network.

We now proceed to formally describe this MDP model. To do so, we first state a number of assumptions regarding the network under consideration:

- (1) The network is divided into a number of basic administrative units (which we shall call "districts" in the following) which may be grouped in larger units (which we call "regions").
- (2) The highways and roads of the networks may be classified into a number of "categories" (e.g. highways, primary roads, and secondary roads). Furthermore, within each category, we may distinguish, if desired, several "types" corresponding to different construction techniques or materials, traffic load ranges, environment, etc. It is assumed that the type of a given pavement section will remain the same throughout the planning horizon and that for any given type the sections of this type are reasonably homogeneous with regard to their dynamic behavior. If previous experience indicates that similar sections react in different ways to the possible maintenance and rehabilitation actions, it is essential to further subdivide the types to achieve the desired homogeneous behavior within each type.

- (3) For every type, it is possible to define a set of condition states which accurately depict serviceability and dynamic behavior.
- (4) For every state of every type, the set of possible maintenance and rehabilitation alternatives is well-defined and non-empty, i.e., there is always one action (which may be a "do nothing" or "routine maintenance" action) which may be performed.
- (5) In each period, exactly one M&R action (which may be the "do nothing" or "routine maintenance" action) is to be performed on each section of the network. Note that if one wishes to allow more than one action to be performed on a section during a period, this may be accomplished by defining new actions which correspond to combinations of the basic actions.
- (6) The planning horizon (which may be infinite) is fixed and is divided into a number of equal-length periods (usually one year).
- (7) For every type ℓ , every pair (s', s) of states of type ℓ , every action a which is allowed in state s' , the probability that a pavement section in state s' will be in state s at the beginning of the next period $p_{s's}^{\ell a}$ is known, stationary (independent of time), and independent of location (district).

To simplify the exposition, let us introduce the following notation:

- D , the set of districts;
- R , the set of regions;
- D_r , the set of districts in region r , $r \in R$;
- C , the set of categories;
- L , the set of types;
- L_c , the set of types for category c , $c \in C$;
- S_ℓ , the set of states for type ℓ , $\ell \in L$;

- A, the set of M&R actions;
- $A_{\ell s}$, the set of allowable actions for state s of type ℓ , $\ell \in L$, $s \in S_{\ell}$;
- T, the planning horizon limit (if finite);
- $b_{\ell s}^d$, the number of lane-miles of type ℓ in state s in district d at the beginning of the planning horizon, $d \in D$, $\ell \in L$, $s \in S_{\ell}$.

We may now state the model in the finite and the infinite planning horizon cases.

2.1 Finite horizon model

In this model, we wish to determine the optimal actions which should be performed on the sections of the network starting at the beginning of period 1 up to the end of period T. As stated earlier, this is an aggregate formulation, and it is therefore impossible to state these actions relatively to specific sections. Instead, the decision variables are defined in accordance with the section grouping scheme which has been chosen. More specifically, for each period t , we want to know the number of lane-miles of sections of type ℓ in state s in district d over which action a should be performed and we denote by $x_{\ell s}^{adt}$ the corresponding decision variable. Our model must not only allow for the choice of optimal actions, it must also provide us with a consistent description of the network. This is achieved through a set of **dynamic constraints** which link the decision variables from one period to the next. To ease the understanding of these constraints, let us first introduce a set of auxiliary variables $y_{\ell s}^{dt}$ which will be used to denote "the expected number of lane-miles of sections of type ℓ in state s in district b at the beginning of period t ".

Let us first note that since we have assumed that exactly one M&R action is performed on every section in each period (assumption 5), we must

have for every possible combination of pavement type, condition state, district and period:

$$\sum_{a \in A_{ls}} x_{ls}^{adt} = y_{ls}^{dt}, \quad l \in L, s \in S_l, d \in D, t \in \{1, \dots, T\}. \quad (1)$$

Consider now what will happen if we perform action a in period $(t-1)$ on $x_{ls'}^{ad(t-1)}$ lane-miles of sections of type l in condition state s' in district d . According to assumption 7, each of these sections will end up being in state s with probability $p_{s's}^{la}$. We can therefore expect $(p_{s's}^{la} x_{ls'}^{ad(t-1)})$ lane-miles of these sections to be in state s at the beginning of period t . If we sum up these quantities over all possible states s' and all possible actions, we obtain the expected number of lane-miles of type l in condition state s in district d at the beginning of period t , i.e., y_{ls}^{dt} .

$$\sum_{s' \in S_l} \sum_{a \in A_{ls'}} p_{s's}^{la} x_{ls'}^{ad(t-1)} = y_{ls}^{dt}, \quad l \in L, s \in S_l, d \in D, t \in \{2, \dots, T\}. \quad (2)$$

By subtracting Equation (2) from Equation (1), we may now link directly the decision variables in period $(t-1)$ with those of period t , without using the y_{ls}^{dt} :

$$\sum_{a \in A_{ls}} x_{ls}^{adt} - \sum_{s' \in S_l} \sum_{a \in A_{ls'}} p_{s's}^{la} x_{ls'}^{ad(t-1)} = 0, \quad l \in L, s \in S_l, d \in D, t \in \{2, \dots, T\}. \quad (3)$$

For $t=1$, we have $y_{ls}^{d1} = b_{ls}^d$ and we obtain the following constraint:

$$\sum_{a \in A_{ls}} x_{ls}^{ad1} = b_{ls}^d, \quad l \in L, s \in S_l, d \in D. \quad (4)$$

The set of equations (3) together with the set of equations (4) constitute the dynamic constraints of the model. Note that for each period and for each district, we have one equation for every possible condition state of every pavement type. This means that for a large network and a long planning horizon we may have a fairly large number of these constraints (for instance, a 10-district network with 5 pavement types with an average of 4 states by type will yield 1,000 dynamic constraints over a five-period planning horizon).

Given any linear objective function defined by coefficients f_{ls}^{adt} for all values of a , d , t , l and s , the finite horizon model consists of optimizing (i.e., minimizing or maximizing) the objective function

$$Z = \sum_{t=1}^T \sum_{d \in D} \sum_{l \in L} \sum_{s \in S_l} \sum_{a \in A_{ls}} f_{ls}^{adt} x_{ls}^{adt} \quad (5)$$

subject to the dynamic constraints and to non-negativity constraints on the x_{ls}^{adt} .

2.2 Infinite horizon model

In this model, we try to determine an optimal systematic policy (repeated in each period) under which the network would remain (on the average) in the same desirable state from period to period. This is to say that if one were to consider a sequence of periods t , $t+1$, $t+2$, ..., one would have $x_{ls}^{adt} = x_{ls}^{ad(t+1)} = x_{ls}^{ad(t+2)} = \dots$, for all l , s , a , and d . We may thus drop the "t" superscript on the decision variables to obtain new decision variables x_{ls}^{ad} . With these new variables the dynamic constraints (3) are easily rewritten as

$$\sum_{a \in A_{ls}} x_{ls}^{ad} - \sum_{s' \in S_l} \sum_{a \in A_{ls'}} p_{s's}^{la} x_{ls'}^{ad} = 0, \quad l \in L, s \in S_l, d \in D. \quad (6)$$

To account for the total number of lane-miles of each type in each district, we add a new set of constraints which is a role similar to that of Equations (4) in the previous formulation:

$$\sum_{s \in S_\ell} \sum_{a \in A_{\ell s}} x_{\ell s}^{ad} = \sum_{s \in S_\ell} b_{\ell s}^d, \quad \ell \in L, d \in D. \quad (7)$$

It should be pointed out that some constraints in the set (5) are redundant and that they should be dropped. It is also possible that the model may breakdown if the types and states are not properly defined. Such occurrence is not possible in the finite horizon model.

2.3 Additional constraints

As indicated before, the linear programming formulation of the MDP model (in the finite or infinite planning horizon version) allows for the addition of arbitrary linear constraints to the model. Such constraints could include budget constraints, performance constraints, bounds on the number of lane-miles over which some actions can be performed, etc. For instance, in the finite horizon model, if the cost of performing action a on a lane-mile of type ℓ in state s is $c_{\ell s}^{at}$ in period t and if one desires to impose an upper budget limit of B^{dt} for every district d in period t , then the corresponding budget constraints could be written as:

$$\sum_{\ell \in L} \sum_{s \in S_\ell} \sum_{a \in A_{\ell s}} c_{\ell s}^{at} x_{\ell s}^{adt} < B^{dt}, \quad d \in D. \quad (8)$$

Further examples of such constraints will be illustrated in section

3. THE FORMAL DESCRIPTIVE ENVIRONMENT

The objective of the formal descriptive environment is to permit a user who is unfamiliar with mathematical programming techniques to fully describe planning problems within the optimization framework previously defined.

It is interesting to note that the problem description may be broken down into two parts: the first part consists of a description of the highway and road network under consideration, i.e., of its logical (categories, types, states, actions) and administrative structure, as well as of all the relevant parameters (transition matrices, initial state, costs, commonly used performance measures, etc.); the second part is concerned with scenario objectives, etc.). To formalize this conceptual division, we have developed a descriptive environment which is made up of two languages: a network description language and a scenario specification language. It should be stressed that the level of technical sophistication required to set up network descriptions is much higher than that for scenario specification. It would thus be possible within an agency to have one person who is more technically knowledgeable being in charge of setting up and maintaining various network description files while the rest of the staff would only have to learn and understand scenario specification to take advantage of the DSS.

We now briefly describe the two languages.

3.1 The network description language

The network description language is a formal language for setting up network description files within the DSS. It specifies the exact format and sequence of elements within such a file. The structure of such a file is as follows:

- network identification block: network name, date, length of description horizon;

- geographic block: number of regions and districts, listing of districts within regions;
- category structure: listing of names of categories;
- type and state structure: for each category, listings of all types with associated condition states;
- action structure: listing of all possible actions with their names and the subset of condition states for which they are allowed;
- function structure: a list of linear functions stating for each one its name, whether it applies to the action variables (the x variables of the previous section) or to the dummy state variables (the y variables) and all of its coefficients;
- initial state structure: a listing of the b parameters of the previous section;
- transition matrix structure: a listing of the p parameters of the model.

As can be deduced from this enumeration, setting up a network description file is a substantial task. To somewhat alleviate this burden, we have tried using compact notations (e.g. for specifying groups of coefficients at one time) whenever possible.

3.2 The scenario specification language

The scenario specification language is a formal language for setting up scenario specification files within the DSS. The general structure of those files is fairly simple since it consists of one line for defining the length of the planning horizon for the scenario, one line for stating the objective function of the model, a few lines for specifying the linear constraints to be added to the basic model, and eventually a "target" specification for the network at the end of the planning horizon.

Within the scope of this language, all functions that are used are always referred to by their mnemonic names (which have been defined in the network description file).

The key feature of this language is the syntax of the CONSTRAINT statement which allows for the specification of whole families of similar constraints by single occurrences of the statement, and for complex patterns in the definition of index scope within those families.

To illustrate this, we now present a few examples.

4. EXAMPLES

Let us consider a highway and road network divided into 10 districts (numbered from 1 to 10) grouped in 3 regions (numbered from 1 to 3) with 3 categories of highways and roads (highways, primary roads, and secondary roads) and 4 possible actions (patching, resurfacing, do-nothing, and crack sealing — CRACKSEAL). For each category there is a single type and 6, 5, and 4 condition states respectively have been defined for highways, primary roads, and secondary roads. Let us also assume that 4 functions have been defined for that network in the network description file:

- COST which gives for each action the average cost per lane-mile taking into account category and condition state;
- UNACCEPT which is a 0-1 function stating whether a given condition state is considered unacceptable for the category in question;
- PCI which gives on a 0-100 scale the average pavement condition index for each state;
- MILEAGE which is a dummy function taking the value 1 for every action and state.

To minimize the total cost of the maintenance and rehabilitation program for that network over a four-period planning horizon, a very short scenario specification file would suffice:

- (1.1) NETWORK EXAMPLE
- (1.2) HORIZON 4
- (1.3) OBJECTIVE MINIMIZE COST.

To solve this simple scenario, 600 dynamic constraints must be generated by the system (one constraint for each period-district-state combination: $4 \times 10 \times (6+5+4)$).

One could also specify scenarios which include very complex constraint sets. Consider the following, for example:

- (2.1) NETWORK EXAMPLE
- (2.2) HORIZON 4
- (2.3) OBJECTIVE MINIMIZE UNACCEPT
- (2.4) CONSTRAINT COST $< 500,000$
- (2.5) CONSTRAINT COST EACH PERIOD $* < 150,000$
- (2.6) CONSTRAINT COST EACH DISTRICT $* > 20,000$
- (2.7) CONSTRAINT MEAN PCI HIGHWAYS EACH PERIOD $* > 75$
- (2.8) CONSTRAINT MILEAGE CRACKSEAL PERIOD 1,2 < 500
- (2.9) CONSTRAINT MILEAGE CRACKSEAL EACH PERIOD 3,4 < 400
- (2.10) CONSTRAINT MEAN UNACCEPT EACH CATEGORY $*$
EACH REGION $* EACH PERIOD $* < 0.1$.$

Since we have the same network and horizon as before, there will be 600 dynamic constraints, but we will have many other constraints:

- statement (2.4) generates a global budget constraint of \$500,000 for the whole M&R program;
- period budget constraints of \$150,000 are enforced by statement (2.5) (4 constraints over all);

- statement (2.6) imposes a lower bound of \$20,000 on total expenses for each of the 10 districts (10 constraints);
- in statement (2.7), we ask that the mean PCI (computed over the whole network) of highways be at least 75 during each of the periods (4 constraints);
- statements (2.8) and (2.9) correspond to a policy of limiting crack sealing to no more than 500 lane-miles in the first two periods (1 constraint) and to no more than 400 lane-miles in each of the last two periods (2 constraints);
- the proportion of unacceptable highways and roads is upper-bounded to 10% (0.1) for each category-region-period combination by statement (2.10) (36 constraints).

It should also be noted that the objective function has been changed: we are minimizing the total lane-mileage of unacceptable highways and roads over the planning horizon.

5. COMPUTER IMPLEMENTATION

Under the proposed DSS approach, the complete solution process of a strategic planning problem is made up of the following steps:

- (1) setting up the network description and scenario specification files;
- (2) translating the user-representation of the problem into a mathematical statement in an adequate format;
- (3) solving the optimization problem;
- (4) reporting results of the optimization routine in a user-friendly format.

Since there exists a large number of linear programming packages available on the market, we decided that we would not develop new software for this purpose, but rather use one of the existing packages: XMP of R.E. Marsten (5).

Up to now, we have developed a program which constructs an XMP input file from both network description and scenario specification files, a validation program for scenario specification files which verifies that those files are set up in accordance with the formal language and which detects and flags erroneous statements, and simple procedure for translating back XMP output into a form which is understandable.

6. FUTURE DEVELOPMENTS

We intend to keep on developing the component programs required for full implementation of the DSS approach that we have proposed.

In the near future, we intend to focus on three major items within the system:

- a) set-up of the network description and scenario specification files for which we plan to develop interactive entry programs;
- b) better optimization procedures (using mathematical decomposition techniques) to allow for the solution of larger problems;
- c) development of a powerful system which would include tables, plots, graphics, etc., for presenting the results of the optimization software.

ACKNOWLEDGEMENTS

This work was supported by a grant from the Fonds F.C.A.R. of the province of Quebec and from the Ministère des Transports du Québec, and by the Natural Sciences and Engineering Council of Canada.

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